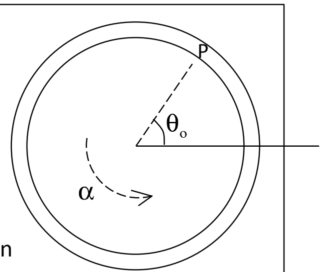
Problem 10.17

A 2.00 meter diameter bike tire rotates from rest with angular acceleration of $4.00\ \text{rad/s}^2$. A point "P" on the rim is located at 57.3°, relative to the horizontal (the x-axis).

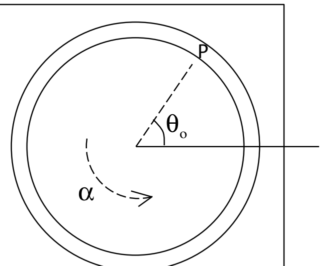


a.) What is its angular speed after 2.00 seconds?

Minor note: As the angular speed is going to be in "radians/second," having the initial angular position in "degrees" won't do. Fortunately, $57.3^\circ = 1.00$ radians.

This is a rotational kinematics problems. Picking the appropriate equation, we get:

$$\begin{aligned}\omega_2 &= \cancelto{0}{\omega_1} + \alpha(\Delta t)\\ &= (4.00\ \text{rad/s}^2)(2.00\ \text{s})\\ &= 8.00\ \text{rad/s}\end{aligned}$$



b.) What is P's translational speed after 2.00 seconds?

$$\begin{aligned}v &= R\omega_2\\ &= (1.00\ \text{m/rad})(8.00\ \text{rad/s})\\ &= 8.00\ \text{m/s}\end{aligned}$$

c.) What is P's total acceleration?

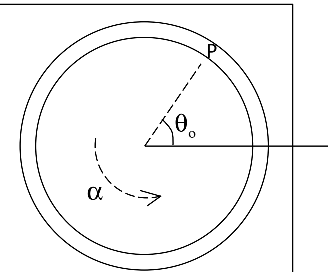
There are two accelerations happening at "P," a tangential acceleration associated with α and a centripetal acceleration. First, the tangential acceleration:

$$\begin{aligned}a_t &= R\alpha\\ &= (1.00\ \text{m/rad})(4.00\ \text{rad/s}^2)\\ &= 4.00\ \text{m/s}^2\end{aligned}$$

And the centripetal:

$$\begin{aligned}a_c &= \frac{v^2}{R}\\ &= \frac{(8.00\ \text{m/s})^2}{(1.00\ \text{m})}\\ &= 64.0\ \text{m/s}^2\end{aligned}$$

So the net acceleration will be:

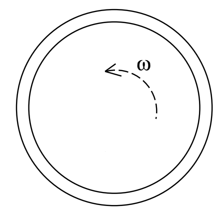


$$\begin{aligned}\vec{a} &= a_c\hat{r} + a_t\hat{\theta}\\ &= (64.0\ \text{m/s}^2)\hat{r} + (4.00\ \text{m/s}^2)\hat{\theta}\end{aligned}$$

If this was my problem, I'd be happy with this. Unfortunately, the text's Solution Manual puts the net acceleration in a polar notation, soooo, using what we know about relating *unit vector* to *polar notation*, we can write:

$$\begin{aligned}\vec{a} &= (64.0\ \text{m/s}^2)\hat{r} + (4.00\ \text{m/s}^2)\hat{\theta}\\ &= \left((64.0\ \text{m/s}^2)^2 + (4.00\ \text{m/s}^2)^2\right)^{1/2} \measuredangle \tan^{-1}\left(\frac{4.00}{64.0}\right)\\ &= (64.1\ \text{m/s}^2)\measuredangle 3.58^\circ\end{aligned}$$

d.) What is the angular position at t = 2.00 seconds?



Another rotational kinematics problem:

$$\begin{aligned}\theta_2 &= \theta_1 + \cancelto{0}{\omega_1}(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2\\ &= (1.00\ \text{rad}) + \frac{1}{2}(4.00\ \text{rad/s}^2)(2.00\ \text{s})^2\\ &= 9.00\ \text{rad}\end{aligned}$$